ON CALCULATING THE DYNAMIC CHARACTERISTICS OF TUBULAR HEATERS

M. A. Fialkov and T. A. Fialkova

UDC 536.27

Following is presented an analytical procedure for calculating the dynamic characteristics of tubular heaters with an intermediate heat carrier.

In many branches of industry one finds a wide use of tubular heaters with water as the heat carrier circulating in a closed system and, in turn, heated by steam in a mixed-phase boiler.

The use of an intermediate heat carrier distinguishes this apparatus from jacket-type heaters with steam passed directly between the tubes, and this feature determines its peculiar dynamic characteristics.

In order to calculate the dynamic characteristics of such an apparatus, we will make use of partial differential equations describing the transient heat transfer which occurs here. It is possible to introduce the following simplifications: the heated product flows uniformly, the thermal flux along the walls and the heat storing capacity of the heat-exchanger wall are negligibly small [1], the physical parameters of the heated product and the heat transfer coefficients are constant and equal to their mean values over the duration of the process; the temperature of the heating water in the circulation system is constant and equal to the arithmetic mean of its entrance and exit temperatures.

In accordance with the heat balance condition, we have

$$mc\frac{dt_1}{d\tau}+kl\int_0^L (t_1-t_2)\,dx=G(i''-t_k), \tag{1}$$

$$\frac{\partial t_2}{\partial \tau}+v\frac{\partial t_2}{\partial x}=\frac{kF}{Lf\rho_p c_p}(t_1-t_2). \tag{2}$$

Using a set of new variables

$$\theta_1=\frac{t_1-t_1^0}{t_{ex}^0},\quad \theta_2=\frac{t_2-t_2^0}{t_{ex}^0},\quad X=\frac{x}{L},\quad \vartheta=\frac{v}{L}\tau,\ \Delta G=G-G_0,$$

we have

$$\frac{d\theta_1}{d\vartheta}+a_1\int_0^1(\theta_1-\theta_2)\,dX=a_2\Delta G, \tag{3}$$

$$\frac{\partial\theta_2}{\partial\vartheta}+\frac{\partial\theta_2}{\partial X}=b(\theta_1-\theta_2), \tag{4}$$

where

$$a_1=\frac{kFL}{mcv};\quad a_2=\frac{(i''-t_k)L}{mcv};\quad b=\frac{kF}{W_p}.$$

In this way, the dynamics of the apparatus will be uniquely defined by the values of two similarity criteria a and b, while the exit temperatures of the heated product are

S. M. Kirov Institute of Agronomy, Omsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 18, No. 5, pp. 860-864, May, 1970. Original article submitted June 27, 1969.

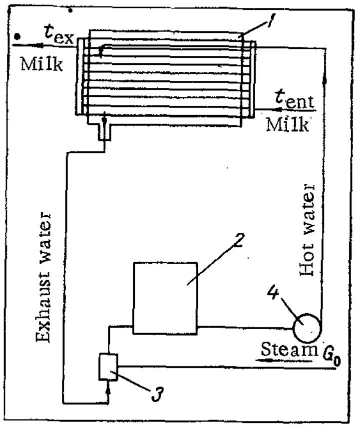

Fig. 1. Schematic diagram of a PTU-5M tubular pasteurizing plant: 1) lower water section of the pasteurization plant; 2) boiler; 3) injector; 4) pump.

$$\theta_{ex} = \frac{t_{ent} - t^0_{ex}}{t^0_{ex}} = \Phi\left(\frac{kF}{W_p};\ \frac{kFL}{mcv};\ \frac{v\tau}{L};\ \frac{(i - t_h)L}{mcvt^0_{ex}}\ \Delta G\right). \tag{5}$$

In order to determine Φ, we find the solution to (3), (4). The initial conditions for

$$\vartheta = 0 \quad X > 0; \quad \theta_2(0) = 0,$$

$$\frac{\partial\theta_1}{\partial\vartheta} = 0; \ \theta_1(X;\ 0) = 0,$$

$$\frac{\partial\theta_1(X;\ 0)}{\partial\vartheta} = 0,$$

are

$$X = 0 \quad \vartheta > 0; \quad \theta_2(0;\ \vartheta) = \theta$$

$$\Delta G = g(\vartheta).$$

By transformation into the time coordinate, we obtain for given initial conditions:

$$pT_1(p) + a_1 \int_0^1 [T_1(p) - T_2(X;\ p)]\, dX = a_2 g(p), \tag{6}$$

$$\frac{dT_2(X;\ p)}{dX} + (p + b)\, T_2(X;\ p) = bT_1(p). \tag{7}$$

From (7) and the boundary conditions, considering that $T_1(p)$ is independent of the space coordinate, we obtain

$$T_2(X;\ p) = T_{ex}\exp[-(p + b)X] + \frac{bT_1(p)}{p + b}(1 - \exp[-(p + b)X]). \tag{8}$$

Insertion of T_2 into (6) and integration over the specified limits will yield

$$T_1 = \frac{a_2(p + b)^2 g(p) + a_1(p + b)[1 - \exp(-p - b)]\, T_{ex}(p)}{p(p + b)(p + a_1 + b) + a_1 b[1 - \exp(-p - b)]}. \tag{9}$$

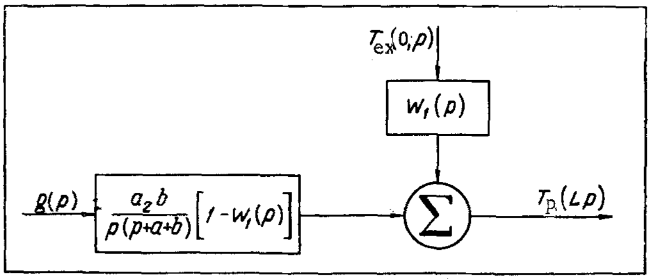

Fig. 2. Structural block diagram of the heater.

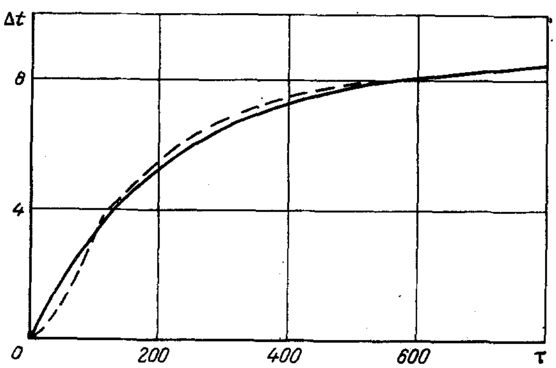

Fig. 3. Transient response of a tubular heater.

Expression (9) describes the temperature dynamics of the heating water as a function of the steam rate and of the heated product temperature at the entrance.

Inserting T_1 from (9) into (8), we obtain for the heated product temperature at the exit

$$T_p(p) = \frac{M(p)T_{ent}}{N(p)} + \frac{Q(p)}{N(p)}g(p) = W_1(p)T_{ent} + W_2(p)g(p), \tag{10}$$

where

$$M(p) = p(p+b)(p+a_1+b)\exp(-p-b) + a_1b\left[1-\exp(-p-b)\right],$$
$$N(p) = p(p+b)(p+a_1+b) + a_1b\left[1-\exp(-p-b)\right],$$
$$Q(p) = a_2b(p+b)\left[1-\exp(-p-b)\right].$$

The transfer function

$$W_1(p) = \frac{p(p+b)(p+a_1+b)\exp(-p-b) + a_1b\left[1-\exp(-p-b)\right]}{p(p+b)(p+a_1+b) + a_1b\left[1-\exp(-p-b)\right]} \tag{11}$$

relates the exit temperature and the entrance temperature of the heated product at a constant steam rate. The transfer function

$$W_2(p) = \frac{a_2b(p+b)\left[1-\exp(-p-b)\right]}{p(p+b)(p+a_1+b) + a_1b\left[1-\exp(-p-b)\right]} = \frac{a_2b}{p(p+a+b)}\left(1-W_1(p)\right) \tag{12}$$

establishes the dependence of the heater exit temperature on the steam rate.

In this way, the structure of the apparatus as a regulated system can be shown schematically as in Fig. 2.

Of most practical interest is the study of temperature dynamics at the exit from the apparatus in response to a perturbation along the heat carrier duct. In order to simplify the computations, we approximate the transcendental transfer function W_2 by its first-order term, i.e., we assume that

$$W_2 \approx \frac{K_{sys}}{Tp+1}. \tag{13}$$

Since the approximate transfer function is equal to the exact transfer function under steady-state conditions ($p \to 0$), then

$$K_{sys} = \frac{a_2b}{a_1}.$$

Since the integral evaluations of transient processes according to the exact and according to the approximate transfer functions are equal

$$\lim_{p\to 0}\frac{d}{dp}\frac{K_{sys}}{Tp+1} = \lim_{p\to 0}\frac{d}{dp}W_2(p),$$

hence we have

$$T = \frac{b^2 - a\left[1-b-\exp(-b)\right]}{a_1b\left[1-\exp(-b)\right]}. \tag{14}$$

The use of the described method is shown in the case of a PTU-5M tubular milk heater whose dynamic characteristics have been calculated. Given: heat exchanging surface 2.25 m^2, rate of heated product (milk) flow 2.9 m/sec, production rate 5,000 liters/h, path length of heated product flow 30 m, k = 1980 W/m^2, $i'' = 27\cdot 10^6$ J/kg, $i_c = 325\cdot 10^3$ J/kg, $c_p = 3.9\cdot 10^3$ J/kg·°K, c = 4.168 J/kg·°K, $\rho_p = 1.030$ kg/m^3, mass of heating medium (water) 150 kg.

According to (13), the exit temperature during a step change in the steam rate is

$$\theta_{ex} = K_{sys}\left[1-\exp\left(-\frac{v\tau}{TL}\right)\right]\Delta G. \tag{15}$$

The curve shown in Fig. 3 was calculated by Eq. (15) for the transient response to $\Delta G = 0.02$ kg/sec. The test curve corresponding to the same perturbation is shown by a dashed line.

NOTATION

c is the specific heat of water, J/kg·°K;
m is the mass of the circulating heating water, kg;

t_1	is the temperature of the heating water, °K;
t_2	is the temperature of the heated product, °K;
τ	is the time, sec;
k	is the heat transfer coefficient, W/m^2;
l	is the length of the flow cross-section perimeter, m;
x	is the space coordinate, m;
G	is the steam rate, kg/sec;
i''	is the enthalpy of the heating steam, J/kg;
i_c'	is the enthalpy of the condensate, J/kg;
L	is the length of the heated product flow path through the apparatus, m;
v	is the velocity of the heated product flow through the apparatus, m/sec;
F	is the heat exchanging surface, m^2;
f	is the cross-section area of the heated product in the apparatus, m^2;
c_p	is the specific heat of the heated product, J/kg·°K;
ρ_p	is the density of the heated product, kg/m^3;
t_1^0	is the steady-state temperature of the heating water, °K;
t_2^0	is the steady-state temperature of the heated product, °K;
t_{ex}^0	is the steady-state temperature of the heated product at the exit from the apparatus, °K;
G_0	is the steady-state steam rate, kg/sec;
$W_p = vf\rho_p c_p$	is the water equivalent of the heated product;
t_{ent}	is the temperature of the heated product at the entrance to the apparatus, °K;
K_{sys}	is the gain of the regulated system.

LITERATURE CITED

1. G. D. Rabinovich, Theory of Heat Calculations for Regenerative Heat Exchangers [in Russian], Izd. Akad. Nauk BSSR (1963).